第 43 卷 第 4 期
2022 年 7 月

华侨大学学报(自然科学版)
Journal of Huaqiao University (Natural Science)

Vol. 43 No. 4
Jul. 2022

DOI:10.11830/ISSN.1000-5013.202103008

线性离散时不变系统的分数阶相位校正迭代学习控制

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摘要: 针对带有随机干扰的线性离散时不变系统,提出一种分数阶相位校正迭代学习控制算法.设计一种新型相位超前校正与分数阶迭代学习控制相结合的迭代学习控制(ILC)学习律.基于频域分析方法,得到分数阶相位校正迭代学习控制在算法开、闭环两种情况下的频域收敛条件.结果表明:文中算法显著提高了 ILC 跟踪误差的收敛速度和收敛精度,具有先进性和有效性.

关键词: 分数阶;相位校正;迭代学习控制;收敛精度

中图分类号: TP 273 **文献标志码:** A **文章编号:** 1000-5013(2022)04-0518-08

Fractional-Order Phase Correction Iterative Learning Control of Linear Discrete Time-Invariant Systems

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Abstract: A fractional-order phase correction iterative learning control algorithm is proposed aiming at linear discrete time-invariant systems with random disturbances. A new iterative learning control (ILC) learning law combining phase advance correction and fractional-order iterative learning control is designed. Base on the frequency domain analysis method, the frequency domain convergence conditions of the fractional-order phase correction iterative learning control algorithm in both open and closed loops are obtained. The results show that the proposed algorithm can significantly improve the convergence speed and convergence precision of ILC tracking errors, and it is advanced and effective.

Keywords: fractional-order; phase correction; iterative learning control; convergence precision

迭代学习控制(ILC)是针对有限时间内具有重复运行特性被控系统的一种有效控制方法[1].经过三十余年的发展,ILC 已成功应用于单轮式移动机器人[2]、机器人操纵器[3]、工业打印机[4]、原子力显微镜[5]和柔性微型飞行器[6]等研究领域.目前,ILC 的研究成果多集中于整数阶控制系统或整数阶的 ILC

收稿日期: 2021-03-04
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基金项目: 国家自然科学基金资助项目(61203369);福建省中青年教育科研基金资助项目(JA15037)

学习算法[7-14]. 然而，在实际工业生产中，分数阶控制系统更符合实际要求[15]，故研究分数阶的迭代学习控制(FO-ILC)具有理论意义和现实意义. Li 等[16]针对线性时变系统，提出带有初始状态更新的 FO-ILC 算法，通过 D^{α} 型分数阶学习律更新每次迭代的初始状态，实现变初始状态迭代学习控制. Zhao 等[17]针对线性时变系统，提出带有初始状态更新和系统输入更新的 FO-ILC 算法，该方法有效提高了 ILC 误差收敛速度. Lü 等[18]针对分数阶多智能体系统的协同一致性跟踪问题，提出分布式 FO-ILC 算法. Li[19]针对分数阶线性时不变系统，提出一阶和二阶的分数阶 PID 型 ILC 策略. Liu 等[20]对分数阶微分系统提出脉冲补偿 ILC. FO-ILC 与其他控制策略结合的混合控制方法取得了较大的研究进展[21-24].

在 ILC 设计中，被控系统容易出现高频段的相位滞后[25]. 为了解决该问题，文献[26-27]提出基于连续系统和离散系统的相位超前迭代学习控制(LPL-ILC)方案，通过引入简单的线性相位超前补偿环节，解决被控系统出现高频段相位滞后的问题，扩大了系统可学习的带宽. Moore 等[28]针对离散系统引入可变增益. 潘雪等[29]提出一种分数线性相位超前补偿迭代学习控制方法，将线性相位超前补偿环节由整数幂改为分数幂，并在算法实现过程中利用拉格朗日插值法近似逼近分数幂. 这类基于分数阶 ILC 的设计方法是间接的[30-31]，ILC 离散化算法的引入会增大 ILC 设计的复杂度. 为了进一步提高控制效果，本文提出一种基于线性离散时不变系统的分数阶相位校正迭代学习控制(FOPC-ILC)算法.

1　基础知识

Grünwald-Letnikov 分数阶积分定义为

$$\left.\begin{aligned}{}_{t_0}J_t^{\alpha}f(t) &= \lim_{h\to 0} h^{\alpha}\sum_{j=0}^{\left[\frac{t-t_0}{h}\right]}\begin{bmatrix}\alpha\\ j\end{bmatrix}f(t-jh),\\ \begin{bmatrix}\alpha\\ j\end{bmatrix} &= \left[\frac{\alpha(\alpha+1)\cdots(\alpha+j-1)}{j!}\right].\end{aligned}\right\}\tag{1}$$

Grünwald-Letnikov 分数阶微分定义为

$$\left.\begin{aligned}{}_{t_0}D_t^{\alpha}f(t) &= \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-t_0}{h}\right]}(-1)^j\binom{\alpha}{j}f(t-jh),\\ \binom{\alpha}{j} &= \frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}.\end{aligned}\right\}\tag{2}$$

式(1)，(2)中：t_0 为初始时刻；t 为随机时刻；h 为时间步长；$j=1,2,\cdots$；$D^{\alpha}(\cdot)$为 α 阶微分，$\alpha\in(0,1)$；[·]表示取整.

由式(2)可推导出分数阶微分数值表达式为

$${}_{t_0}D_t^{\alpha}f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-t_0}{h}\right]}(-1)^j\binom{\alpha}{j}f(t-jh)\approx\frac{1}{h^{\alpha}}\sum_{j=0}^{\infty}\xi_j^{\alpha}f(t-jh).\tag{3}$$

式(3)中：计算精度为 $o(h)$；$\xi_j^{\alpha}=(-1)^j\binom{\alpha}{j}$，其可由递推公式得到，即

$$\xi_0^{\alpha}=1,\quad \xi_j^{\alpha}=\left(1-\frac{1-\alpha}{j}\right)\xi_{j-1}^{\alpha}.$$

式(3)的 Z 域表达式为

$$Z({}_{t_0}D_t^{\alpha}f(t)) = Z\left(\frac{1}{h^{\alpha}}\sum_{j=0}^{\infty}\xi_j^{\alpha}f(t-jh)\right)=\left(\frac{1}{h^{\alpha}}\sum_{j=0}^{\infty}\xi_j^{\alpha}z^{-jh}\right)F(z) = (h^{-\alpha}\sum_{j=0}^{\infty}\xi_j^{\alpha}z^{-jh})F(z).\tag{4}$$

式(3)中：$F(z)$为 $f(t)$的 Z 域表达式；z^{-jh} 为 j 次频域分量.

2　分数阶相位校正迭代学习控制

迭代学习控制的主要目的是通过 ILC 学习律修正系统输入，使系统输出能跟踪到期望输出. 文中

讨论的单输入单输出(SISO)线性离散时不变系统方程为

$$\left.\begin{aligned}&x_k(n+1)=\boldsymbol{A}x_k(n)+\boldsymbol{B}u_k(n)+w_k(n),\\&y_k(n)=\boldsymbol{C}x_k(n)+\boldsymbol{D}u_k(n).\end{aligned}\right\}\tag{5}$$

式(5)中：n 为系统运行时间，$n\in[0,N]$；k 为迭代次数；$x_k(n)$，$u_k(n)$，$y_k(n)$分别为系统第 k 次迭代的状态、输入和输出；$w_k(n)$为系统第 k 次迭代的随机干扰；$\boldsymbol{A}\sim\boldsymbol{D}$ 均为系数矩阵.

对式(5)两边同时进行 Z 变换，可得系统在 Z 域上表达式，即

$$\left.\begin{aligned}&zX_k(z)=\boldsymbol{A}X_k(z)+\boldsymbol{B}U_k(z)+W_k(z),\\&Y_k(z)=\boldsymbol{C}X_k(z)+\boldsymbol{D}U_k(z).\end{aligned}\right\}\tag{6}$$

式(6)中：z 为频域分量；$X_k(z)$，$U_k(z)$，$Y_k(z)$分别为系统第 k 次迭代的状态、输入和输出的 Z 域表达式；$W_k(z)$为 第 k 次迭代的随机干扰 Z 域表达式.

线性离散时不变系统满足以下 3 个假设.

1) 假设 1. 每一次迭代的初始状态都相同，且 $x_k(0)=0$.

2) 假设 2. $|\Delta W_k(z)|=|W_{k+1}(z)-W_k(z)|\leqslant\delta$，$\delta$ 为一常数.

3) 假设 3. 对于系统(5)，(6)，存在唯一的期望系统状态，分别为 $x_{\mathrm{d}}(n)$，$X_{\mathrm{d}}(z)$，存在唯一的期望系统输入，分别为 $u_{\mathrm{d}}(n)$，$U_{\mathrm{d}}(z)$，有

$$\left.\begin{aligned}&x_{\mathrm{d}}(n+1)=\boldsymbol{A}x_{\mathrm{d}}(n)+\boldsymbol{B}u_{\mathrm{d}}(n)+w_k(n),\\&y_{\mathrm{d}}(n)=\boldsymbol{C}x_{\mathrm{d}}(n)+\boldsymbol{D}u_{\mathrm{d}}(n),\end{aligned}\right\}\tag{7}$$

$$\left.\begin{aligned}&zX_{\mathrm{d}}(z)=\boldsymbol{A}X_{\mathrm{d}}(z)+\boldsymbol{B}U_{\mathrm{d}}(z)+W_k(z),\\&Y_{\mathrm{d}}(z)=\boldsymbol{C}X_{\mathrm{d}}(z)+\boldsymbol{D}U_{\mathrm{d}}(z).\end{aligned}\right\}\tag{8}$$

式(7)，(8)中：$y_{\mathrm{d}}(n)$为期望系统输出；$Y_{\mathrm{d}}(z)$为期望系统输出的 Z 域表达式.

定义 ILC 跟踪误差表达式为

$$e_k(n)=y_{\mathrm{d}}(n)-y_k(n).\tag{9}$$

式(9)的 Z 域表达式为

$$E_k(z)=Y_{\mathrm{d}}(z)-Y_k(z).\tag{10}$$

2.1 开环控制

根据分数阶微分数值算法给出分数阶相位校正开环 ILC 的收敛条件及其证明过程. 根据式(3)，(4)，提出时域开环学习律，即

$$u_{k+1}(n)=u_k(n)+L\frac{1}{h^{\alpha}}\sum_{j=0}^{n}\xi_j^{\alpha}e_k(n-jh+\lambda)=u_k(n)+LD^{\alpha}(e_k(n+\lambda)).\tag{11}$$

式(11)中：L 为 ILC 学习增益；λ 为超前拍次.

对式(11)进行 Z 变换，可得其 Z 域学习律，即

$$U_{k+1}(z)=U_k(z)+L(z^{\lambda}+h^{-\alpha}\sum_{j=0}^{n}\xi_j^{\alpha}z^{-jh})E_k(z).\tag{12}$$

由式(11)可知：相较于 LPL-ILC 和整数阶 ILC 学习律，FOPC-ILC 学习律增加了一个新的 α 自由度调节因子(α 微分)，能够调节 ILC 的收敛速度. 传统的 D 型学习律由于高阶差分作用，对于高频噪声信号具有放大作用，从而造成系统控制性能降低. 由式(12)可知：当前时刻的输入 $U_{k+1}(z)$ 是由上一次迭代时所有时刻的误差$(z^{\lambda}+h^{-\alpha}\sum_{j=0}^{n}\xi_j^{\alpha}z^{-jh})E_k(z)$组成，$h^{-\alpha}\sum_{j=0}^{n}\xi_j^{\alpha}z^{-jh}$ 等效于有限冲击响应低通滤波器，因而能够有效滤除高频噪声.

定理 1 针对线性离散时不变系统(式(5)，(6))，在假设 1～3 的条件下，采用 FOPC-ILC 开环学习律(式(11)，(12))，如果 ILC 学习增益 L 满足

$$\left|1-H_p(z)L(z^{\lambda}+h^{-\alpha}\sum_{j=0}^{n}\xi_j^{\alpha}z^{-jh})\right|<1,\tag{13}$$

那么，$\lim\limits_{k\to+\infty}|E_k(z)|=\sum\limits_{i=0}^{k-1}\left[\left|1-H_p(z)L\cdot(z^{\lambda}+h^{-\alpha}\sum\limits_{j=0}^{n}\xi_j^{\alpha}z^{-jh})\right|^{k-1}|\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_{k-i}(z)|\right]$. 其中，$\boldsymbol{I}$

为单位矩阵；$\xi_j^\alpha=(-1)^j\binom{\alpha}{j}$为式(4)的系数；$H_p(z)$为式(5)的传递函数；$\Delta W_{k-i}(z)$满足假设 2，且 $|\Delta W_{k-i}(z)|\leqslant\delta$.

证明：由式(8)可得

$$Y_k(z)=(\boldsymbol{C}(z-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D})U_k(z)+\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}W_k(z)=H_p(z)U_k(z)+\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}W_k(z). \tag{14}$$

式(14)中：

$$H_p(z)=\boldsymbol{C}(z-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}. \tag{15}$$

联合式(8)，(10)，可得

$$\begin{aligned}E_{k+1}(z)=&E_k(z)+(Y_k(z)-Y_{k+1}(z))=\\&E_k(z)+H_p(z)(U_k(z)-U_{k+1}(z))+\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}(W_k(z)-W_{k+1}(z))=\\&E_k(z)-H_p(z)(U_{k+1}(z)-U_k(z))-\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_k(z).\end{aligned} \tag{16}$$

将式(12)代入式(16)，可得

$$E_{k+1}(z)=E_k(z)-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})E_k(z)-\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_k(z). \tag{17}$$

对式(17)两边同时取模，并由复数模的性质可得

$$|E_{k+1}(z)|\leqslant\left|1-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})\right||E_k(z)|+|\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_k(z)|. \tag{18}$$

对式(18)进行 k 次迭代，可得

$$\begin{aligned}|E_{k+1}(z)|\leqslant&\left|1-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})\right|^k|E_1(z)|+\\&\sum_{i=0}^{k-1}\left|1-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})\right|^{k-1}\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_{k-i}(z).\end{aligned} \tag{19}$$

因此，当 $\left|1-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})\right|<1$，有

$$\lim_{k\to+\infty}|E_k(z)|=\sum_{i=0}^{k-1}\left[\left|1-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})\right|^{k-1}|\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_{k-i}(z)|\right],$$

则 ILC 误差收敛于随机扰动界，定理 1 得证.

2.2 闭环控制

闭环学习律为

$$u_{k+1}(n)=u_k(n)+L\frac{1}{h^\alpha}\sum_{j=0}^{n}\xi_j^\alpha e_{k+1}(n-jh+\lambda)=u_k(n)+LD^\alpha(e_{k+1}(n+\lambda)). \tag{20}$$

式(20)的 Z 域表达式为

$$U_{k+1}(z)=U_k(z)+L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})E_{k+1}(z). \tag{21}$$

定理 2　针对线性离散时不变系统(式(6)，(7))，在假设 1～3 的条件下，采用 FOPC-ILC 闭环学习律(式(20)，(21))，如果满足

$$\left|\frac{1}{1+H_p(z)(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|<1, \tag{22}$$

那么，

$$\begin{aligned}\lim_{k\to+\infty}|E_k(z)|=\sum_{i=0}^{k-1}&\left[\left|\frac{1}{1+H_p(z)(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|^{k-1}\times\right.\\&\left.\left|\frac{\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_{k-i}(z)}{1+H_p(z)(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|\right].\end{aligned}$$

证明：与定理 1 的证明过程相同，将式(21)代入式(16)，可得

$$E_{k+1}(z)=E_k(z)-H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})E_{k+1}(z)+$$
$$\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_k(z)\frac{1}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}E_k(z)+$$
$$\frac{\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_k(z)}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}. \tag{23}$$

对式(23)两边同时取模,可得

$$|E_{k+1}(z)|\leqslant\left|\frac{1}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right||E_k(z)|+$$
$$\left|\frac{\boldsymbol{C}(zI-\boldsymbol{A})^{-1}\Delta W_k(z)}{1+H_p(z)L\cdot(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|. \tag{24}$$

对式(24)进行 k 次迭代,可得

$$|E_{k+1}(z)|\leqslant\left|\frac{1}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|^k|E_1(z)|+$$
$$\sum_{i=0}^{k-1}\left|\frac{1}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|^{k-1}\times$$
$$\left|\frac{\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}W_{k-i}(z)}{1+H_p(z)L\cdot(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}\xi_j^\alpha z^{-jh})}\right|. \tag{25}$$

因此,当 $\left|\frac{1}{1+H_p(z)L(z^\lambda+h^{-\alpha}\sum_{j=0}^{n}z^{-jh})}\right|<1$ 时,有

$$\lim_{k\to+\infty}|E_k(z)|=\sum_{i=0}^{k-1}\left[\left|\frac{1}{1+H_p(z)L(z^\lambda+hT^{-\alpha}\sum_{j=0}^{n}z^{-jh})}\right|^{k-1}\times\right.$$
$$\left.\left|\frac{\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\Delta W_{k-i}(z)}{1+H_p(z)L(z^\lambda+hT^{-\alpha}\sum_{j=0}^{n}z^{-jh})}\right|\right],$$

则 ILC 跟踪误差收敛于随机扰动,定理 2 得证.

3 仿真结果

通过实例,仿真验证文中算法的有效性.被控系统的传递函数为

$$H_p(z)=\boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}=\frac{z-0.8}{z^2-1.4z+0.45}. \tag{26}$$

期望系统输出为

$$y_{\mathrm{d}}(n)=1-\exp(-n). \tag{27}$$

相关参数设置如下:初始输入 $u_0(n)=0$;$w_k(t)$采用随机数模拟随机扰动;相位校正阶数 $\gamma=2.5$;根据定理 1,2,确定 ILC 学习增益 $L=0.6$.采用误差均方根(RMS)量化跟踪误差,有

$$\mathrm{RMS}=\sqrt{\sum_{i=0}^{N}e_k(i)^2/(N+1)}. \tag{28}$$

文中算法和文献[29]的开、闭环学习律的跟踪轨迹和跟踪误差的仿真结果,如图 1～8 所示.图 1～8 中:$y(n)$为系统输出;$y_5(n)$为系统迭代 5 次的输出,其他表示类似.

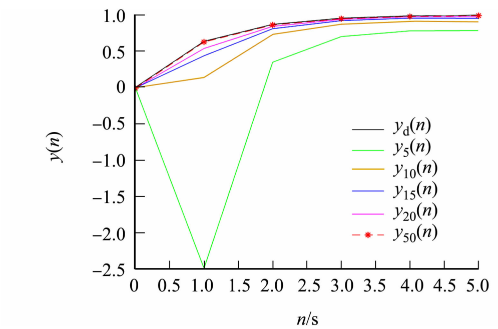

图 1　文中算法开环学习律的跟踪轨迹

Fig. 1　Tracking trajectory of open loop learning law of proposed algorithm

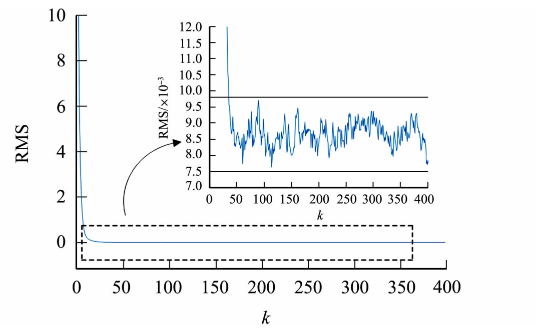

图 2　文中算法开环学习律的跟踪误差

Fig. 2　Tracking error of open loop learning law of proposed algorithm

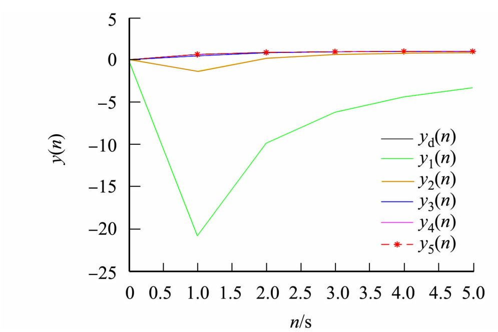

图 3　文中算法闭环学习律的跟踪轨迹

Fig. 3　Tracking trajectory of closed loop learning law of proposed algorithm

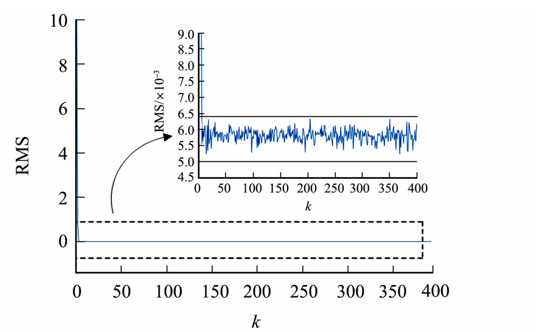

图 4　文中算法闭环学习律的跟踪误差

Fig. 4　Tracking error of closed loop learning law of proposed algorithm

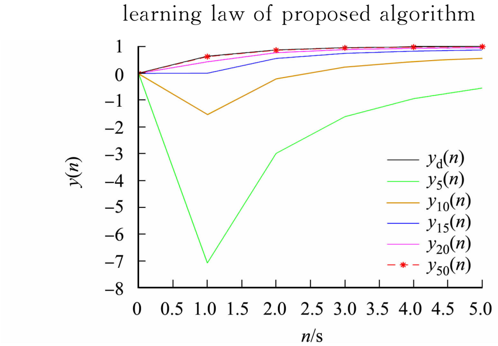

图 5　文献[29]开环学习律的跟踪轨迹

Fig. 5　Tracking trajectory of open loop learning law of reference [29]

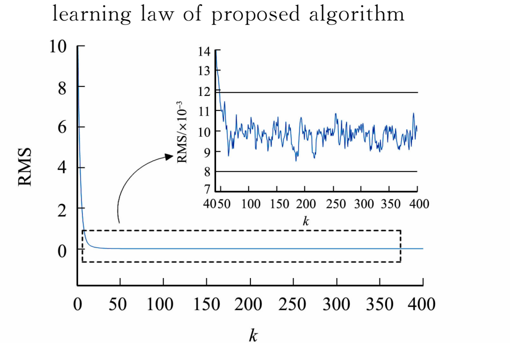

图 6　文献[29]开环学习律的跟踪误差

Fig. 6　Tracking error of open loop learning law of reference [29]

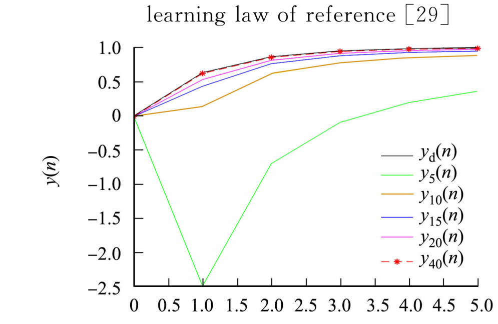

图 7　文献[29]闭环学习律的跟踪轨迹

Fig. 7　Tracking trajectory of closed loop learning law of reference [29]

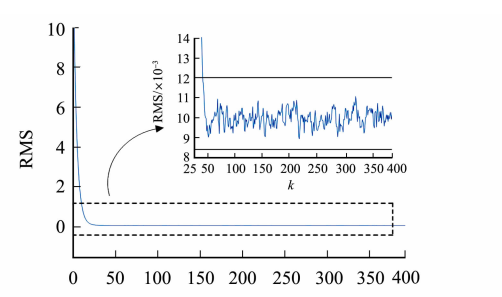

图 8　文献[29]闭环学习律的跟踪误差

Fig. 8　Tracking error of closed loop learning law of reference [29]

由图 1～8 可知：无论采用文中算法开、闭环学习律，还是采用文献[29]开、闭环学习律（LPL-ILC），在一定的迭代次数后，都能使系统输出跟踪到期望轨迹（期望输出），从而保证跟踪误差的单调收敛. 由

图 2 可知:文中算法开环学习律跟踪误差约稳定收敛于 0.009 0,误差波动范围为 0.007 5～0.009 8.由图 4 可知:文中算法闭环学习律跟踪误差约稳定收敛于 0.005 8,误差波动范围为 0.005 0～0.006 4.由图 6,8 可知:文献[29]开、闭环学习律跟踪误差约稳定收敛于 0.010 0,误差波动范围分别为 0.008 0～0.011 9,0.008 4～0.012 0.由图 2,4,6,8 可知:文中算法的跟踪误差收敛精度更高,且误差波动范围更小.由图 1,3,5,7 可知:采用文中算法开环学习率、文中算法闭环学习率、文献[29]开环学习律、文献[29]闭环学习律,分别在迭代次数为第 50,5,50,40 次时,跟踪误差达到稳定状态,系统输出达到期望轨迹;采用闭环学习律的收敛速度快于开环学习律的;采用文中算法闭环学习律的收敛速度最快.

综上可知,文中算法不仅解决了系统跟踪过程无法单调收敛的问题,而且提高了跟踪误差的收敛速度和收敛精度,其中,文中算法闭环学习律对跟踪误差的收敛速度和收敛精度的提高效果更为显著.

4 结束语

基于线性离散时间系统,提出一种分数阶相位校正迭代学习控制算法.相较于 FO-ILC 算法,文中算法避免了 ILC 算法的离散化过程,可保证算法理论分析和实现过程的一致性.由仿真结果可知,文中算法具有先进性和有效性,可提高跟踪误差的收敛速度和收敛精度,特别地,FOPC-ILC 闭环学习律对跟踪误差的收敛速度和收敛精度的提高效果更为显著.

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